

## Engineering Mathematics

A Foundation for Electronic, Electrical, Communications and Systems Engineers

### 15 Applications of integration

#### INTRODUCTION

**Currents and voltages** often vary with time. Engineers may wish to know the average value of such a current or voltage over some particular time interval. The average value of a time-varying **function** is defined in terms of an **integral**. An associated quantity is the **root mean square (r.m.s.)** value of a fonction. The **r.m.s.** value of a current is used in **the calculation of the power dissipated by a resistor**.

### Example 15.2 Sawtooth waveform

Consider the sawtooth waveform shown in Figure 15.2.

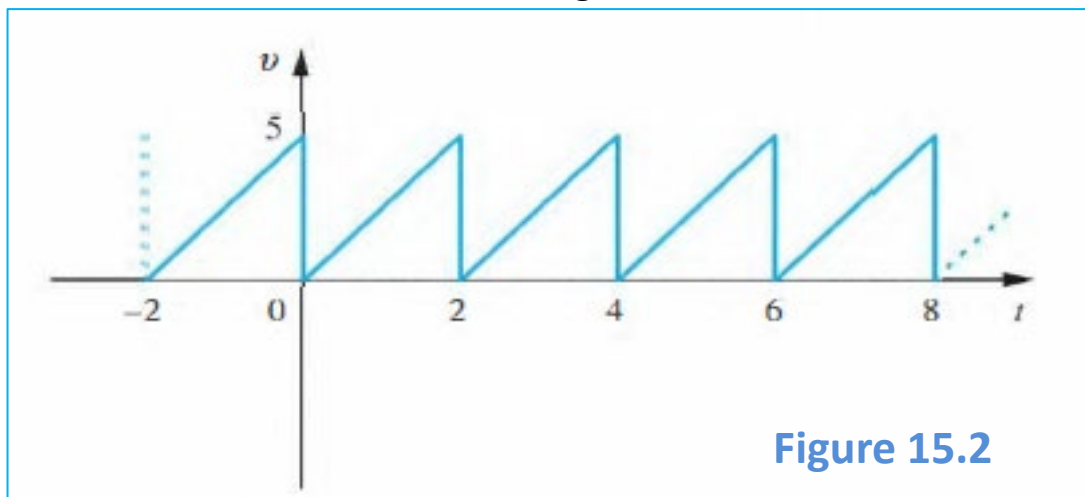


Figure 15.2

**A sawtooth waveform.**

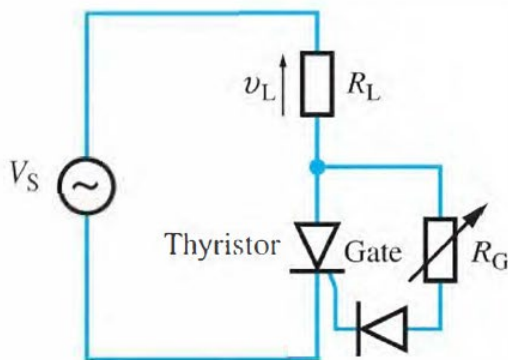
**Calculate the average value of this waveform over a complete period.**

## Engineering Mathematics

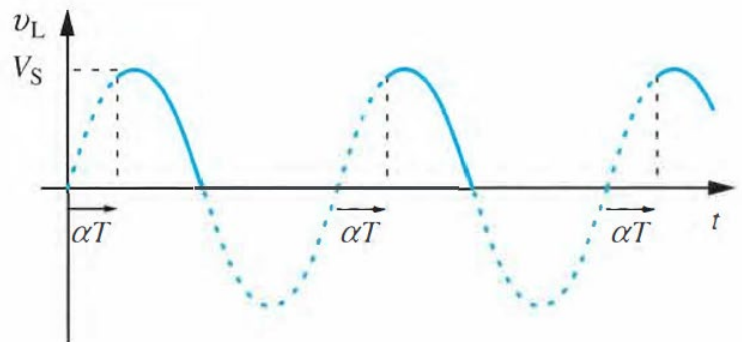
### Example 15.3 A thyristor firing circuit

Figure 15.3 shows a simple circuit to control the voltage across a load resistor,  $R_L$ . This circuit has many uses, one of which is to adjust the level of lighting in a room. The circuit has an a.c. power supply with peak voltage,  $V_S$ . The main control element is the thyristor. This device is similar in many ways to a diode. It has a very high resistance when it is reverse biased and a low resistance when it is forward biased. However, unlike a diode, this low resistance depends on the thyristor being 'switched on' by the application of a gate current. The point at which the thyristor is switched on can be varied by varying the resistor,  $R_G$ . Figure 15.4 shows a typical waveform of the voltage,  $v_L$ , across the load resistor.

The point at which the thyristor is turned on in each cycle is characterized by the quantity  $\alpha T$ , where  $0 \leq \alpha \leq 0.25$  and  $T$  is the period of the waveform. This restriction



**Figure 15.3**  
A thyristor firing circuit.



**Figure 15.4**  
Load voltage waveform.

on  $\alpha$  reflects the fact that if the thyristor has not turned on when the supply voltage has peaked in the forward direction then it will never turn on.

**Calculate the average value of the waveform over a period and comment on the result.**